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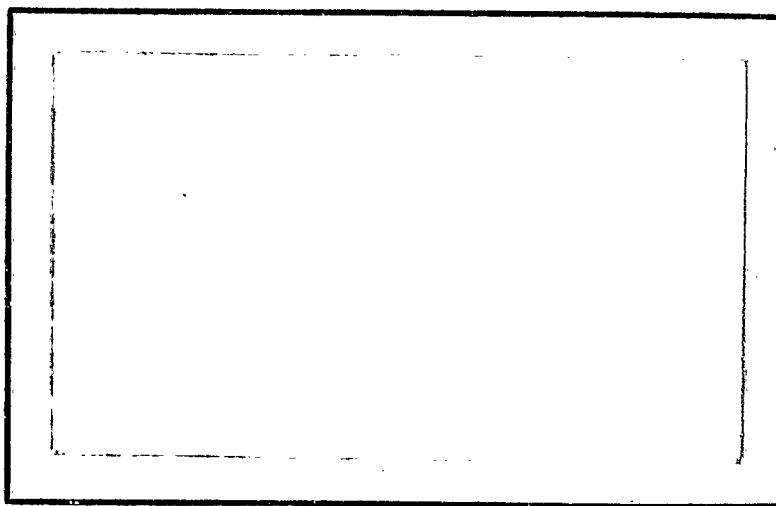
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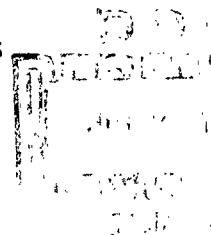
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Bottom Pressure Fluctuations due
to Standing Waves in a Deep, Two-
Layer Ocean.

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by

Henry A. Kierstead

Technical Report
submitted to the Office of Naval Research
under Contract No. N6onr-27701 (NR-083-004).

August, 1951

Approved for Distribution



Director

Bottom Pressure Fluctuations due to Standing Waves
in a Deep, Two-Layer Ocean

by

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and
Brown University, Providence, R. I.

Abstract

The pressure fluctuation in a deep ocean, due to short-period surface waves, is calculated by means of a simple physical model. It is shown, as was pointed out earlier by Longuet-Higgins, that the pressure under a standing wave varies at twice the frequency of the wave and with an amplitude proportional to the square of the wave amplitude, and inversely proportional to the wave length, but that the pressure under a progressive wave is constant.

A similar calculation of the pressure fluctuation under a standing internal wave gives similar results, but the amplitude is diminished by the factor ($\Delta \rho / \rho$).

Numerical calculations, using typical wave amplitudes, show that pressure fluctuations of about 30 cm. may occasionally be expected under 6-second surface waves, but that internal waves, because of their large wave lengths, will have little effect on the pressure in the deep ocean.

The purpose of this paper is to investigate the fluctuations of pressure at the ocean bottom due to surface and internal waves. Longuet-Higgins (1950), by a rigorous

analysis of the case of short-period waves, has shown that progressive waves cause no pressure fluctuations in the deep water, but that in the presence of standing waves the bottom pressure varies with twice the frequency of the surface waves. We will not repeat his calculations, but will show how his results can be derived in a more elementary manner, and apply the same method to calculate the bottom pressure fluctuations due to short-period internal waves.

Under long-period waves the pressure can be calculated with sufficient accuracy from the height of water, on the basis of the usual hydrostatic assumptions, so this case requires no special consideration

1. Free Surface Waves

We consider water of density ρ which is motionless except in the neighborhood of the surface. Let (x, y, z) be Cartesian coordinates with the origin on the mean surface and the z axis directed vertically downward, and let P and V be the pressure and velocity vector. We will further assume that ρ is a constant at great depths. Then the equation of motion is

$$\frac{dV}{dt} = -\text{grad} (P/\rho - gz) \quad (1)$$

Therefore there can be no pressure gradients, except for the hydrostatic term, in the deep water where \underline{V} is zero. But \underline{P} may vary with time providing it is the same at all points in the deep water. We utilize this fact to calculate \underline{P} by considering the motion of the column of water between the surface and the plane $\underline{z} = \underline{z}'$, and of such great lateral extent that the motion of water through its boundaries is negligible compared to the total mass. We denote by $\underline{\zeta}$ the elevation of the free surface above the mean surface.

As has been pointed out by Longuet-Higgins, the center of gravity of this water column will rise and fall as a result of wave motion on the surface, except in the case of pure progressive waves. Vertical motions of the center of gravity can result only from the vertical components of external forces acting on the water mass. These forces are due to gravity, the external atmospheric pressure, and the pressure \underline{P} at $\underline{z} = \underline{z}'$. Let \underline{z}_0 be the depth of the center of gravity of the water column, \underline{M} its mass, \underline{A} its horizontal cross-sectional area, \underline{P}' the dynamic pressure at $\underline{z} = \underline{z}'$, and \underline{P}_0 the surface (atmospheric) pressure.

Then, by definition,

$$\underline{P}' = \underline{P} - \underline{Mg}/\underline{A} - \underline{P}_0 \quad (2)$$

$$\underline{M} = \int_{\underline{A}} \int_{-\underline{\zeta}}^{\underline{z}'} \rho \, d\underline{z} \, d\underline{S} \quad (3)$$

$$z_0 = \frac{1}{M} \int_A \int_{-j}^{z'} \rho z dz dS \quad (4)$$

By Newton's second law,

$$M \frac{d^2 z_0}{dt^2} = \int_A (P_0 - P) dS + Mg \quad (5)$$

$$= A(P_0 - P) + Mg = -AP' \quad (6)$$

if we assume the surface pressure to be constant and take z' great enough so that P is essentially independent of x and y . We will further assume for the moment that P is constant.

Then

$$P' = -\frac{M}{A} \frac{d^2 z_0}{dt^2} \quad (7)$$

$$= -\frac{P}{A} \frac{d^2}{dt^2} \int_A \int_{-j}^{z'} z dz dS \quad (8)$$

$$= \frac{P}{2A} \frac{d^2}{dt^2} \int_A (j^2 - z'^2) dS \quad (9)$$

$$= \frac{P}{2} \frac{d^2}{dt^2} \bar{j}^2 \quad (10)$$

where $\bar{j}^2 = \frac{1}{A} \int_A j^2 dS$

This is the result obtained by Longuet-Higgins in a more rigorous analysis of the wave motion.

2. Internal Waves

We will now apply the same technique to a two-layer ocean. Taking the origin of coordinates at the free surface as in Section 1, let the density and average thickness of the upper layer be ρ' and h' , and those of the lower layer be ρ and h . The height of the interface above the plane $z = +h$ will be denoted by ζ , and the elevation of the free surface above $z = 0$ by ζ' . Equation (4) then becomes

$$z_0 = \frac{1}{2M} \int_A [\rho \{z'^2 - (h - \zeta)^2\} + \rho' \{(h - \zeta)^2 - \zeta'^2\}] dS \quad (11)$$

$$P' = -\frac{1}{2A} \frac{d^2}{dt^2} \int_A [\rho \{z'^2 - \zeta^2 + 2h\zeta - h^2\} + \rho' \{\zeta^2 - \zeta'^2 - 2h\zeta + h^2\}] dS \quad (12)$$

$$= \frac{1}{2} \frac{d^2}{dt^2} \{ \rho \bar{\zeta}^2 + \rho' (\bar{\zeta}'^2 - \bar{\zeta}^2) \} \quad (13)$$

$$= \frac{1}{2} \frac{d^2}{dt^2} \{ \rho' \bar{\zeta}'^2 + \bar{\zeta}^2 (\rho - \rho') \} \quad (14)$$

since $\bar{\zeta} = \bar{\zeta}' = 0$ by definition

and $\frac{d^2 h}{dt^2} = \frac{d^2 z'}{dt^2} = 0$

3. Pressure Variations under Harmonic Waves

Equations (10) and (14) give the relation between the bottom pressure and the shape of the surface. However, this relation is more clearly seen if the surface motion is represented by harmonic waves. Because of Fourier's theorem this results in no loss of generality.

We assume, then, that the surface displacement can be represented by the Fourier series

$$\zeta = \sum_{n=1}^{\infty} \{A_n \sin(k_n x + l_n y + \omega_n t + \delta_n)\} \quad (15)$$

where $\omega_n^2 = g(k_n^2 + l_n^2)^{1/2}$

Then
$$\begin{aligned} \zeta^2 = & \sum_{n=1}^{\infty} A_n^2 \sin^2(k_n x + l_n y + \omega_n t + \delta_n) \\ & + 2 \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_n A_m \sin(k_n x + l_n y + \omega_n t + \delta_n) \\ & \times \sin(k_m x + l_m y + \omega_m t + \delta_m) \end{aligned} \quad (16)$$

$$= \frac{1}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} [A_n \sin(k_n x + l_n y + \omega_n t + \delta_n) + A_m \sin(k_m x + l_m y + \omega_m t + \delta_m)] \quad (17)$$

We see that ζ^2 is a sum of terms, each containing a pair of harmonic components. This means physically that the harmonic components interact in pairs, and characterizes the phenomenon as one of the second order. It will be necessary,

therefore, to calculate the bottom pressure fluctuations due to the interaction of two harmonic components. The effect of any arbitrary surface motion can then be calculated by summing over all pairs of functions which occur in the Fourier expansion of ζ . Furthermore, the arbitrary phases δ_n may, without loss of generality, be made to vanish by a suitable choice of the origins of time and space. We therefore take

$$\zeta = A \sin(kx + ly + \omega t) + B \sin(k'x + l'y + \omega't) \quad (18)$$

where

$$\omega^2 = (k^2 + l^2)^{1/2} \quad (19)$$

$$\omega'^2 = (k'^2 + l'^2)^{1/2} \quad (20)$$

\underline{k} , \underline{l} , \underline{k}' , \underline{l}' , are arbitrary wave numbers, and \underline{A} and \underline{B} are arbitrary amplitudes. Then

$$\begin{aligned} \zeta^2 = & A^2 \sin^2(kx + ly + \omega t) + B^2 \sin^2(k'x + l'y + \omega't) \\ & + 2AB \sin(kx + ly + \omega t) \sin(k'x + l'y + \omega't) \end{aligned} \quad (21)$$

But

$$\sin^2(kx + ly + \omega t) = \frac{1}{2} [1 - \cos 2(kx + ly + \omega t)] \quad (22)$$

The first term on the right is independent of t , while the second has an average value of zero. Therefore the third term in (21) is the only one which will contribute to $\frac{d^2}{dt^2} \bar{y}$.

We now use the trigonometric identity

$$\begin{aligned} 2AB \sin(kx+ly+wt) \sin(k'x+l'y+w't) \\ = AB [\cos\{(k-k')x+(l-l')y+(w-w')t\} \\ - \cos\{(k+k')x+(l+l')y+(w+w')t\}] \end{aligned} \quad (23)$$

But the right side of this equation has a mean of zero unless $k = k'$, $l = l'$, or $k = -k'$, $l = -l'$, either of which implies $w = w'$ because of (19) and (20). If $k = k'$, $l = l'$, (23) is replaced by

$$2AB \sin^2(kx+ly+wt) = AB [1 - \cos 2(kx+ly+wt)] \quad (24)$$

with the constant mean AB . This is to be expected, since the center of gravity of a progressive wave moves horizontally, but not vertically.

We find, therefore, that a single progressive wave, or a linear combination of progressive waves with different wave lengths or directions of propagation, will not result in pressure fluctuations in the deep ocean.

On the other hand, if $\underline{k} = -\underline{k}'$, $\underline{l} = -\underline{l}'$, (23)

becomes

$$\begin{aligned} & 2AB \sin(kx + ly + \omega t) \sin(-kx - ly + \omega t) \\ & = AB [\cos 2(kx + ly) - \cos 2\omega t] \end{aligned} \quad (25)$$

Therefore

$$\frac{d^2}{dt^2} \zeta^2 = -AB \frac{d^2}{dt^2} \cos 2\omega t = 4AB \omega^2 \cos 2\omega t \quad (26)$$

and from (10)

$$P' = 2\rho AB \omega^2 \cos 2\omega t = 2\rho g AB (k^2 + l^2)^{1/2} \cos 2\omega t \quad (27)$$

We see that there are no pressure variations unless there exists in the surface waves a pair of harmonic components of equal frequency but opposite directions of propagation, which is the condition for standing waves. In this case the pressure will vary with twice the frequency of the surface waves. This is, of course, to be expected for a second-order effect and is supported by physical reasoning, since the center of gravity of a standing wave rises every half-cycle and reaches its minimum elevation each time the surface passes through the mean surface.

If there are two layers of different density, the situation is more complicated, since there are two modes of vibration, in each of which both surfaces are

in motion, but with different amplitudes. We consider first the high-frequency mode, analogous to free-surface waves, in which the two layers move together. For this mode the frequency is given by the usual relation:

$$\omega'^2 = gk \quad (28)$$

where k is the wave number in the direction of propagation. In view of our previous result that pressure variations arise only from the interaction of a pair of waves travelling in opposite directions, which holds for internal waves as well, we can, without loss of generality, assume the waves to be travelling along the x axis. If we let

$$\eta' = A' \sin(kx + \omega't) + B' \sin(kx - \omega't) \quad (29)$$

then we have at the interface, (Lamb, 1932)

$$\eta = A' e^{-kh'} \sin(kx + \omega't) + B' e^{-kh'} \sin(kx - \omega't) \quad (30)$$

Proceeding as before, but using (14) instead of (10), we readily find

$$P' = 2\omega'^2 A' B' [p' + \Delta p e^{-kh'}] \cos 2\omega't \quad (31)$$

$$= 2gk A' B' [p' + \Delta p e^{-kh'}] \cos 2\omega't \quad (32)$$

where

$$\Delta P = P - P' \quad (33)$$

When the surface layer is deep compared to the wave length ($kh' \gg 1$), we have

$$P' \approx 2gk A'B' P' \cos 2\omega't \quad (32a)$$

the same result as that obtained for a one-layer ocean, which is to be expected since in this case the lower layer is unaffected by the wave motion.

When kh' is small,

$$P' \approx 2gk A'B' [P - \Delta P kh'] \cos 2\omega't \quad (32b)$$

which shows the damping effect of a shallow surface layer.

For the other mode of vibration (Lamb, loc. cit.)

$$\omega^2 = gk \frac{\Delta P}{P \coth kh' + P'} \quad (34)$$

and

$$\frac{y'}{y} = - \frac{\Delta P}{P'} e^{-kh'} \quad (35)$$

Let

$$\zeta = A \sin(kx + \omega t) + B \sin(kx - \omega t) \quad (36)$$

and

$$\zeta' = -A \frac{\Delta \rho}{\rho} e^{-kh'} \sin(kx + \omega t) - B \frac{\Delta \rho}{\rho} e^{-kh'} \sin(kx - \omega t) \quad (37)$$

Using (14) again, we find

$$P' = 2\Delta \rho A B \omega^2 (1 - e^{-kh'}) \cos 2\omega t \quad (38)$$

$$= 2gkAB \left(\frac{\Delta \rho}{\rho} \right)^2 \frac{(1 - e^{-kh'}) \sinh kh'}{\rho \cosh kh' + \rho' \sinh kh'} \cos 2\omega t \quad (39)$$

$$= 2gkAB \left(\frac{\Delta \rho}{\rho} \right)^2 \frac{1 - e^{-kh'} - e^{-2kh'} + e^{-3kh'}}{\rho + \rho' + \Delta \rho e^{-2kh'}} \cos 2\omega t \quad (40)$$

When the surface layer is thin, the effect due to the free surface nearly cancels that due to the interface and (40) becomes

$$P' = 2gAB \left(\frac{\Delta \rho}{\rho} \right)^2 k^3 h'^2 \cos 2\omega t \quad (40a)$$

On the other hand, when kh' is large, the motion of the surface is negligible and the pressure

has its maximum value:

$$P' = 2 g k A B \frac{(\Delta \rho)^2}{\rho + \rho'} \cos 2 \omega t \quad (40b)$$

Comparing (40) with (32), we might expect the effect of internal waves to be small compared to that of surface waves when $\Delta \rho$ is small. But the amplitudes of internal waves are generally larger than those of surface waves by a factor of the order of magnitude of $\Delta \rho / \rho$. The ratio of the two effects will therefore depend on k , and since the wave lengths usually observed in internal waves are much greater than those observed in free-surface waves, the effect of free-surface waves will predominate in the bottom pressure fluctuations, as can be seen by reference to the numerical calculations in Section 4.

One further possibility must be discussed, that is, the interference between a free-surface mode and an interfacial mode of equal wave lengths but different frequencies ω and ω' given by (19) and (20), (with $\underline{z} = 0$). We take

$$\zeta' = A' \sin(kx + \omega't) - B \frac{\Delta \rho}{\rho'} e^{-kh'} \sin(kx + \omega t) \quad (41)$$

$$\zeta = A' e^{-kh'} \sin(kx + \omega't) + B \sin(kx + \omega t) \quad (42)$$

Neglecting terms that are independent of time,
we have

$$\bar{f}^2 = -\frac{KAB\Delta P}{2\pi\rho'} e^{-kh'} \int_0^{2\pi k} [\cos(w'-w)t - \cos\{2kx + (w+w')t\}] dx \quad (43)$$

$$= -A'B \frac{\Delta P}{\rho} e^{-kh'} \cos(w'-w)t \quad (44)$$

$$\bar{f}^2 = A'B e^{-kh'} \cos(w'-w)t \quad (45)$$

$$\rho' \bar{f}^2 + \Delta P \bar{f}^2 = 0 \quad (46)$$

$$\therefore P' = 0 \quad (47)$$

Therefore interaction of the two modes of vibration cannot produce pressure changes in deep water.

4. Application to Bottom Pressure Gauges

In the design of bottom pressure gauges and the interpretation of data obtained from them, it will be useful to have some crude estimates of the periods and amplitudes to be expected. The amplitudes (\underline{G}) may conveniently be expressed in centimeters of water.

Then

$$G = \frac{P'}{\rho g \cos 2\omega t} \quad (48)$$

For the spectrum of surface waves we take a "typical" distribution of wave heights given by Munk (1947). This distribution has a maximum height (crest to trough) of four feet, at a period of 8 seconds. Since we wish to estimate the effects of the largest waves which are commonly encountered, we have arbitrarily doubled the wave heights; that is, we have plotted Munk's wave heights as amplitudes. This spectrum is shown in curve 1 of Figure 1. We have used only that part of Munk's spectrum below a period of one minute, since longer waves do not satisfy the condition for surface waves ($L < 2h$). From (27) (with $\underline{L} = 0$) and (48) we find

$$\underline{G} = 2 \underline{A} \underline{B} k \quad (49)$$

$$= 2 \underline{A}^2 \underline{r} k \quad (50)$$

where $\underline{r} = \underline{B}/\underline{A}$ is the standing-wave ratio. It is difficult to guess a typical value for \underline{r} in the deep ocean. It is probably very small except near a moving storm or an island with a very steep shoreline. In any event, \underline{r} can never exceed unity. We will assume in this calculation that \underline{r} equals one, with the understanding that our

results will represent upper limits to the effects that might be observed.

When the data of curve 1 are substituted for \underline{A} in (47) with $\underline{r} = \underline{1}$, we obtain the values of \underline{G} plotted in curve 2. The greatest pressure fluctuation obtained is 32 cm. of water at a period of 3 seconds. For this curve the periods are those of the bottom pressure, i.e. half of the wave periods.

Typical data on internal waves are more difficult to obtain, but it appears that oscillations of the seasonal thermocline at $\underline{h}' = 200$ meters ($\frac{\Delta p}{p} \sim 3 \cdot 10^{-3}$) occur with periods of about 1000 sec. and amplitudes in the neighborhood of 30 m, while the deep thermocline at about 1000 m ($\frac{\Delta p}{p} \sim 5 \cdot 10^{-4}$) oscillates at tidal periods with amplitudes not exceeding 100 m. The deep ocean pressure fluctuations due to waves on the seasonal thermocline is about 10^{-3} cm. The periods observed in the deep thermocline correspond to wave lengths in excess of 8×10^4 m, much greater than the depth of the ocean, so our method of calculation cannot be used in this case. A simple hydrostatic calculation shows that in this case the first-order pressure variation has an amplitude of about 1.2 cm in phase with the internal wave.

We can conclude, therefore, that a bottom pressure recorder with a sensitivity of, say, 5 cm of water, will record no effects due to internal waves,

and that free-surface waves may possibly cause pressure fluctuations with periods of less than 30 seconds, with a maximum amplitude not exceeding 32 cm of water at a period of three seconds.

Acknowledgments

I should like to express my gratitude to Mr. Henry Stommel and Dr. Arnold B. Arons of the Woods Hole Oceanographic Institution for their valuable advice in the preparation of this paper.

Henry A. Kierstead

References

- Lamb, Horace, 1932: Hydrodynamics, 6th ed., p. 372.
Macmillan.
- Longuet-Higgins, M. S. 1950: A theory of the origin
of microseisms. Phil. Trans. Roy.
Soc., No. 857, 243, pp. 1-35.
- Munk, Walter H., 1947: Tracking storms by forerunners
of swell. Jour. Meteor. No. 2, 4,
pp. 45-57.

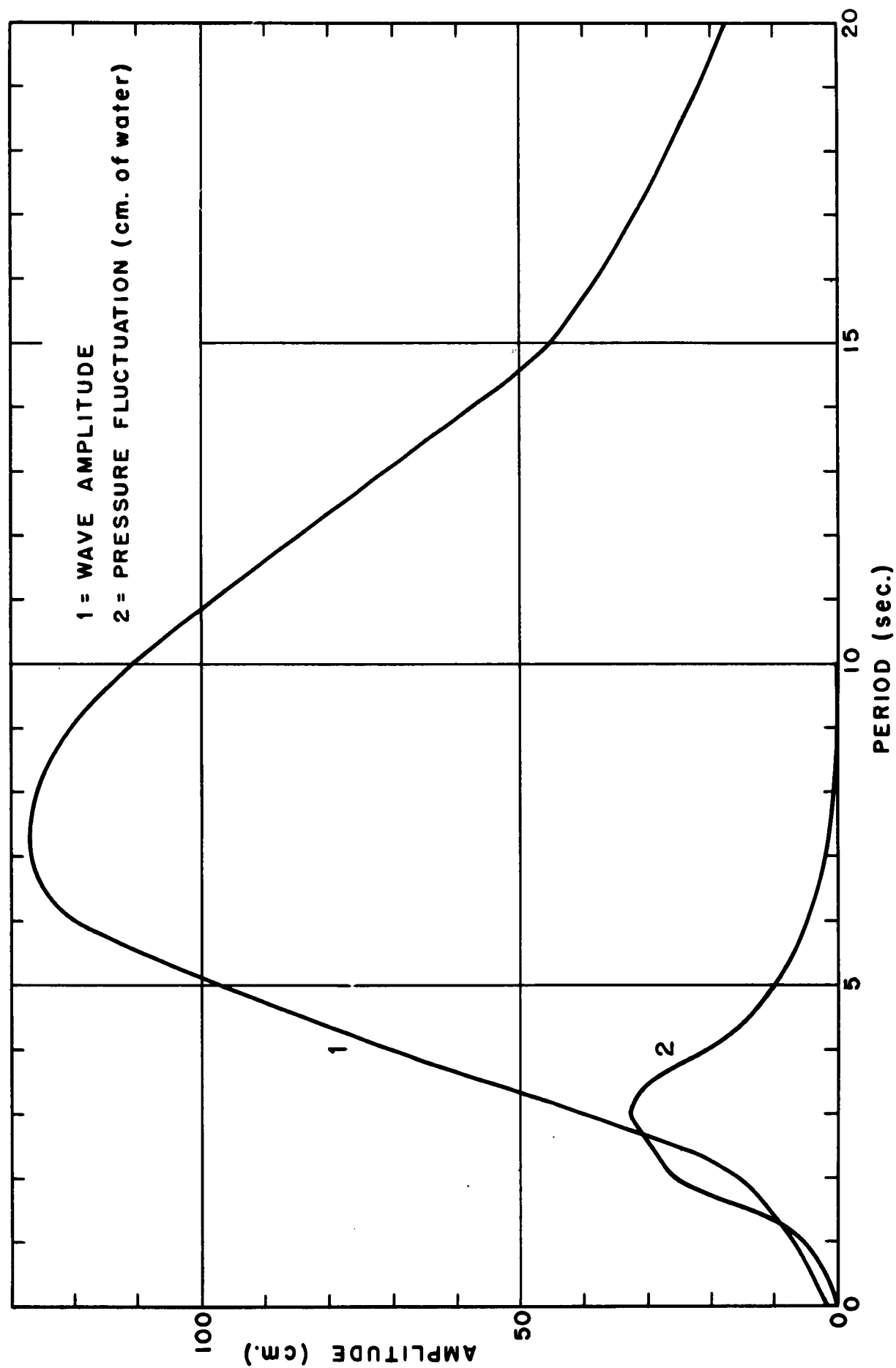


FIG.1 TYPICAL WAVE AMPLITUDES AND PRESSURE FLUCTUATIONS

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